Precision Reduction in Quantum Channels: A Universal Channel Law

Deepinder Sidhu*
Department of Computer Science and Electrical Engineering
University of Maryland, Baltimore County
sidhu@umbc.edu

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Abstract

Quantum channels inevitably degrade coherence and thereby reshape the precision scaling predicted by ideal Heisenberg limits. Rather than attributing this reduction to probe choice, measurement strategy, or architectural details, we present a compact, channel-centric framework in which the impact of noise is captured by a single coherence function governing an L-fold coherent interrogation.

Within this formulation, the channel alone determines both how long ideal Heisenberg scaling can be sustained and where precision saturation must occur. The commonly observed "knee" in noisy-metrology precision curves emerges as a direct and unavoidable consequence of the competition between coherent phase amplification and channel-induced loss of state distinguishability.

By isolating channel effects into a retention function, we introduce simple, dimensionless metrics that quantify (i) the fraction of ideal Heisenberg performance that survives noise and (ii) the remaining advantage over the standard quantum limit. This perspective yields a universal and platform-independent design principle: precision is limited not by how deeply one attempts to interrogate coherently, but by how rapidly the channel erases the information that deeper interrogation seeks to amplify.

1 Introduction

Heisenberg scaling,

$$\Delta\phi \propto \frac{1}{N}, \tag{1}$$

represents the ideal benchmark of quantum metrology [1]. In noiseless settings, coherent interrogation amplifies phase information quadratically with increasing resource count. However, when sensing is mediated by a noisy quantum channel, this ideal scaling inevitably deteriorates as interrogation depth increases.

Optical loss, dephasing, amplitude damping, phase diffusion, and control noise all reduce the distinguishability of quantum states during interrogation. Crucially, this degradation is governed primarily

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by the structure of the quantum channel itself rather than by the microscopic details of the probe state or the measurement strategy [2, 3, 4]. As a result, the breakdown of Heisenberg scaling is a generic feature of noisy quantum sensing rather than a platform-specific anomaly.

The goal of this paper is to address the following question in a concise and platform-independent manner:

How far from ideal Heisenberg scaling must one fall when a quantum channel mediates the sensing process?

Following the coherence-budget viewpoint introduced in Ref. [5], we model channel-induced coherence loss through a *channel coherence function* $\Lambda(L)$, which determines how much of the ideal L^2 coherent amplification survives after L sequential interrogation steps. This channel-centric description isolates the effect of noise at the level of state distinguishability, independent of probe construction or measurement choice.

Relationship to prior resource-partitioning analyses. Previous work [5] analyzes architectural resource partitioning under a fixed total budget $N = ML$ and demonstrates how the saturation knee emerges as a constrained optimum in the (M, L) design space. By contrast, the present paper isolates a different and complementary question: for a fixed quantum channel and with M held constant, how does the channel coherence law $\Lambda(L)$ alone determine the unavoidable reduction from ideal Heisenberg scaling and the channel-limited optimal interrogation depth?

1.1 Interpreting Precision and Channel-Based Metrics

In channel-mediated sensing, a physical quantity—such as phase, frequency, field strength, or time offset—is encoded into a parameter ϕ that accumulates coherently over an interrogation duration. In discrete form, this process is modeled as an L-fold coherent interrogation, where L counts coherent phase-imprinting steps rather than classical repetitions.

Throughout this work, all bounds are evaluated under a consistent interrogation-time convention: either the sensing duration per coherent step is fixed, or time is absorbed uniformly into the resource accounting. Under this assumption, time-dependent prefactors do not affect relative scaling behavior or channel-retention conclusions.

The achievable estimation precision is bounded by the quantum Cramér–Rao inequality,

$$\mathrm{Var}(\hat{\phi}) \geq \frac{1}{F_Q^{\mathrm{eff}}(L)}, \tag{2}$$

where $F_Q^{\mathrm{eff}}(L)$ denotes the effective quantum Fisher information after channel action. The achievable precision is therefore defined as

$$\Delta\phi_{\mathrm{ach}}(L) = \sqrt{\mathrm{Var}(\hat{\phi})} = \frac{1}{\sqrt{F_Q^{\mathrm{eff}}(L)}}. \tag{3}$$

This definition emphasizes that precision degradation occurs prior to measurement: any mechanism that reduces the distinguishability of output states after channel action necessarily increases $\Delta\phi_{\mathrm{ach}}$, even if measurements themselves are ideal.

Two complementary, dimensionless channel-based metrics are used throughout:

- **Channel-retention metric**
$$R(L) \equiv \frac{F_Q^{\rm eff}(L)}{F_Q^{\rm ideal}(L)} = \Lambda(L), \tag{4}$$
which quantifies the fraction of ideal Heisenberg performance preserved after L coherent interrogations.

- **Channel-advantage metric**
$$G(L) \equiv \frac{F_Q^{\rm eff}(L)}{F_Q^{\rm SQL}(L)}, \tag{5}$$
which measures the remaining advantage relative to the standard quantum limit.

The metric $R(L)$ diagnoses fundamental coherence survival, while $G(L)$ determines whether quantum enhancement remains practically deployable. Together, they separate intrinsic channel limitations from operational performance.

Bounds on the Channel Advantage Metric

The impact of a quantum channel on achievable precision can be made explicit by comparing the effective quantum Fisher information to the standard quantum limit (SQL). In the present channel-centric formulation, the effective QFI after an L-fold coherent interrogation takes the reduced Heisenberg form

$$F_Q^{\rm eff}(L) = L^2\Lambda(L), \tag{6}$$

where the factor L^2 represents ideal coherent amplification and $\Lambda(L) \in [0, 1]$ captures the cumulative coherence loss imposed by the channel.

By contrast, the SQL corresponds to independent interrogation steps and scales linearly with interrogation depth, $F_Q^{\rm SQL}(L) \propto L$. The channel-based advantage over the SQL may therefore be written (up to an $\mathcal{O}(1)$ normalization constant common to all strategies) as

$$G(L) \propto L\,\Lambda(L). \tag{7}$$

This expression has a direct physical interpretation. The factor L reflects the linear gain expected from coherent accumulation relative to independent interrogations, while $\Lambda(L)$ quantifies the fraction of that gain that survives channel-induced decoherence. The product $L\Lambda(L)$ therefore measures the net benefit of increasing coherent interrogation depth.

If the channel coherence decays faster than $1/L$—as occurs for exponential, Gaussian, or stronger decay laws—then $L\Lambda(L)$ possesses a finite maximum. Beyond this point, the loss of distinguishability induced by the channel outpaces the ideal coherent gain, and additional interrogation depth reduces rather than improves performance. This behavior underlies the universal appearance of a saturation knee in noisy-metrology curves.

For the representative case of Markovian decoherence,

$$\Lambda(L) = e^{-\gamma L}, \tag{8}$$

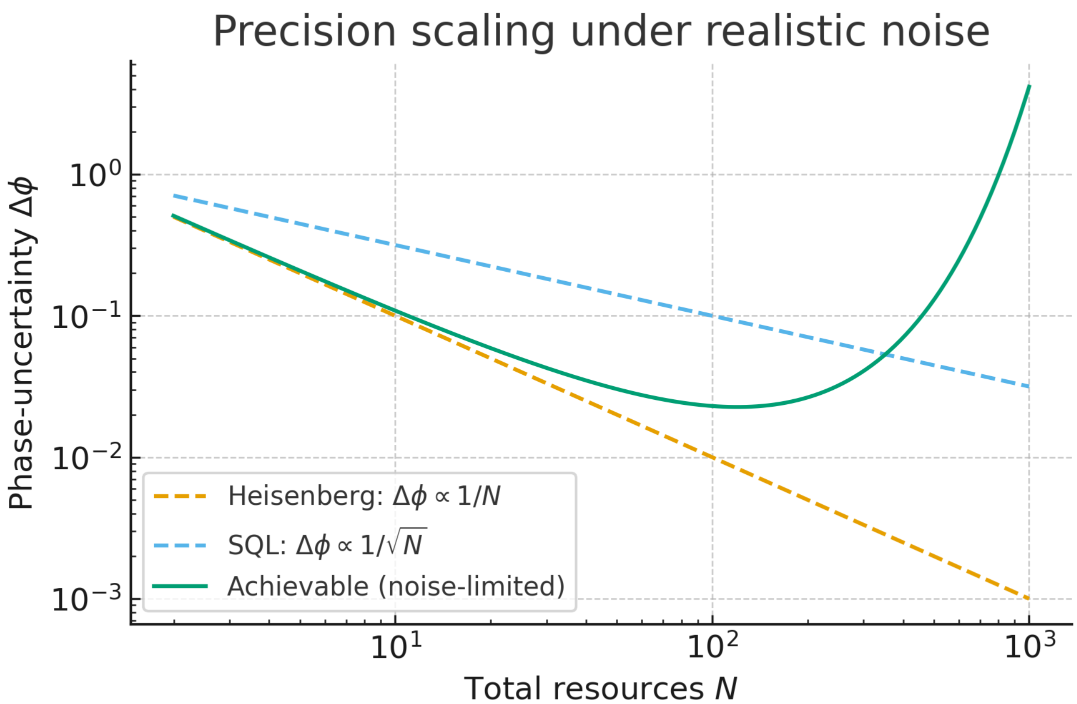

Figure 1: **Universal precision scaling and the channel-induced saturation knee.** Shown are ideal Heisenberg scaling ($\Delta\phi \sim 1/L$), SQL scaling ($\Delta\phi \sim 1/\sqrt{L}$), and the achievable precision $\Delta\phi_{\rm ach}(L)$ in the presence of channel-induced coherence loss. The minimum identifies the channel-determined saturation knee, beyond which additional coherent interrogation degrades rather than improves precision.

the channel-advantage metric reaches its maximum at

$$L^* \simeq \frac{1}{\gamma}, \tag{9}$$

which depends only on the channel decoherence rate and is independent of probe state, measurement strategy, or estimator construction.

Thus, the quantum channel alone determines both whether a sustained advantage over the SQL is possible and the interrogation depth at which that advantage must terminate. The channel-determined maximum of $G(L)$ provides the quantitative origin of the saturation knee observed across noisy quantum-metrology platforms.

2 Parameter Encoding and the Channel-Limited Origin of $F_Q^{\rm eff}(L) = L^2\Lambda(L)$

A sensing parameter ϕ is encoded into a probe state through a unitary transformation of the form

$$U_\phi = e^{-i\phi G}, \tag{10}$$

where G is the Hermitian generator associated with the probe–signal interaction (for example, a number operator in interferometry or a collective spin component in magnetometry). This unitary describes a single coherent parameter-imprinting step.

A coherent interrogation of depth L corresponds to applying the same encoding operation L times without intermediate measurement or decoherence reset,

$$U_{\phi}^{(L)} = e^{-i\phi(LG)}. \tag{11}$$

Physically, L counts the number of coherent phase-accumulation steps or, equivalently, the coherent interrogation duration measured in units of the elementary encoding process.

For ideal interrogation and pure probe states, the generator of the encoding is effectively amplified from G to LG. Since the quantum Fisher information is proportional to the variance of the generator, this coherent amplification produces a quadratic enhancement,

$$F_Q^{\text{ideal}}(L) \propto L^2, \tag{12}$$

which is the fundamental origin of Heisenberg scaling in sequential quantum metrology.

In realistic sensing scenarios, however, the probe does not evolve under the unitary encoding alone. Interaction with the environment during interrogation reduces the distinguishability of the output states and suppresses the usable quantum Fisher information. Rather than modeling this process microscopically, we capture the net effect of the quantum channel through a single, nonnegative *coherence (retention) factor* $\Lambda(L)$, which quantifies the fraction of the ideal L^2 scaling that survives after an L-fold coherent interrogation.

The effective quantum Fisher information may therefore be written as

$$F_Q^{\text{eff}}(L) = L^2\Lambda(L). \tag{13}$$

This expression cleanly separates the roles of encoding and noise. The factor L^2 reflects the ideal coherent amplification dictated solely by the parameter encoding, while $\Lambda(L)$ encapsulates all channel-induced degradation of state distinguishability. Any departure from Heisenberg scaling is therefore attributable entirely to the decay of $\Lambda(L)$ with interrogation depth, rather than to limitations of the encoding mechanism itself.

3 Universal Channel Law

The reduction of achievable precision in noisy quantum metrology follows a universal, channel-determined law that is independent of probe details, measurement strategy, or estimator construction. This law arises from a fundamental competition between two effects: coherent phase accumulation, which ideally grows quadratically with interrogation depth, and channel-induced coherence loss, which progressively suppresses state distinguishability as interrogation proceeds.

For an L-fold coherent interrogation, the effective quantum Fisher information takes the generic form

$$F_Q^{\text{eff}}(L) = L^2\,\Lambda(L), \tag{14}$$

where the channel coherence function $\Lambda(L) \in [0, 1]$ captures the fraction of ideal Heisenberg information retained after L coherent interrogation steps. All channel dependence enters exclusively through $\Lambda(L)$. Consequently, the channel determines not only the rate at which precision degrades, but also whether increasing interrogation depth remains beneficial at all.

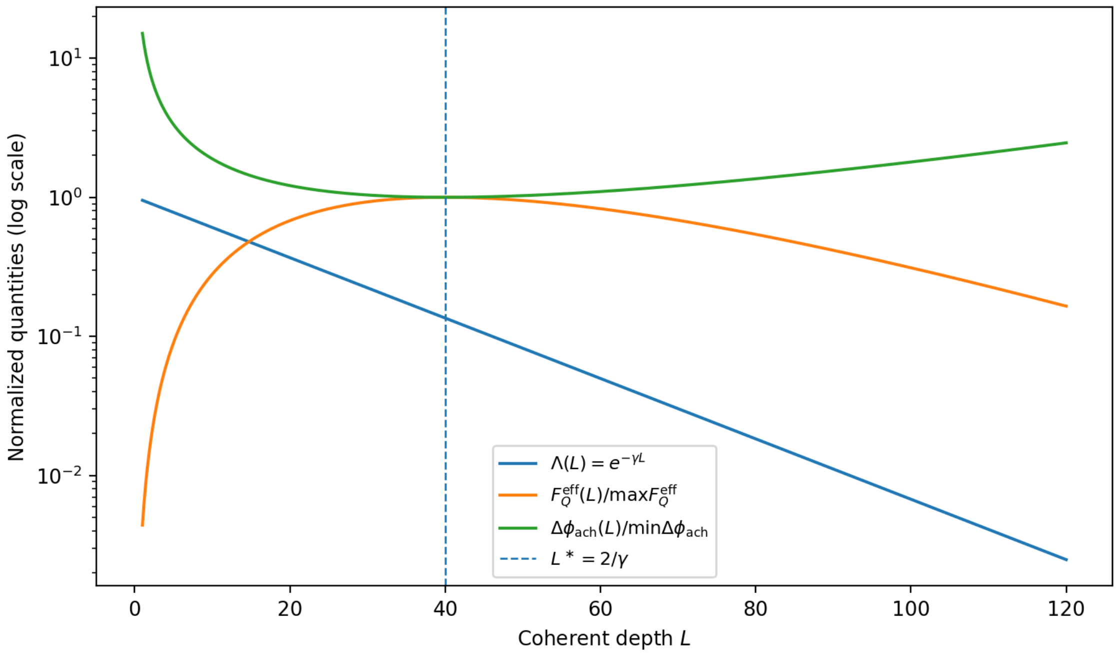

Figure 2: **Universal channel law: coherence, information, and precision on a single axis.** Shown together (normalized for shape comparison) are the channel coherence function $\Lambda(L)$, the effective quantum Fisher information $F_Q^{\mathrm{eff}}(L) = L^2\Lambda(L)$ (normalized by its maximum), and the achievable phase uncertainty $\Delta\phi_{\mathrm{ach}}(L) = 1/\sqrt{F_Q^{\mathrm{eff}}(L)}$ (normalized by its minimum). The channel-determined optimum L^* marks the point beyond which increasing coherent depth converts phase accumulation into decoherence rather than precision gain, producing the saturation knee.

For Markovian channels with exponential coherence decay,

$$\Lambda(L) = e^{-\gamma L}, \tag{15}$$

the retained information initially increases with L due to the ideal L^2 amplification, but eventually decreases once the exponential penalty dominates. Maximizing the effective quantum Fisher information,

$$\frac{d}{dL}\left(L^2 e^{-\gamma L}\right) = 0 \quad \Rightarrow \quad L^* = \frac{2}{\gamma}, \tag{16}$$

reveals a unique, finite optimal interrogation depth set entirely by the channel parameter γ.

This optimal depth L^* defines the channel-limited boundary of useful coherent interrogation. Beyond this point, additional coherent evolution no longer increases phase sensitivity and instead accelerates decoherence, reducing the retained quantum Fisher information. The commonly observed saturation knee in noisy-metrology precision curves is therefore not a plotting artifact or estimator-dependent crossover, but the direct one-dimensional manifestation of this channel-induced optimum.

Crucially, this optimum is fixed at the level of state distinguishability, as quantified by the quantum Fisher information. Its location is therefore independent of probe structure, entanglement strategy, or measurement choice. The saturation knee is not a failure of quantum enhancement, but the natural outcome of optimizing coherent interrogation under a finite channel coherence budget.

Key implications of the universal channel law.

- **Channel-determined termination of Heisenberg scaling.** The existence of a finite optimum L^* implies that asymptotic Heisenberg scaling is unattainable in any channel with sufficiently rapid coherence decay. Although the ideal encoding generates an L^2 enhancement, the channel imposes a hard ceiling on usable coherent depth. Beyond L^*, additional interrogation does not merely yield diminishing returns—it actively reduces the retained quantum Fisher information.
- **Universality across platforms and probe constructions.** Because L^* depends only on the functional form of $\Lambda(L)$, the same saturation mechanism applies uniformly across optical interferometers, atomic clocks, solid-state sensors, and superconducting platforms. Differences between technologies enter only through the effective decay rate γ (or its non-Markovian generalization), not through probe architecture or measurement protocol. In this sense, the universal channel law defines a platform-independent upper bound on coherent interrogation depth and a fundamental ceiling on achievable precision under realistic noise.

4 Conclusions

The reduction from ideal Heisenberg scaling in realistic quantum sensing is dictated entirely by the channel's coherence function $\Lambda(L)$.

This paper demonstrates that precision degradation in quantum metrology is a direct and quantitative consequence of channel-induced coherence loss. The universal form

$$F_Q^{\text{eff}}(L) = L^2\Lambda(L)$$

makes explicit that coherent interrogation depth amplifies information only to the extent that the channel preserves state distinguishability.

The saturation knee commonly observed in noisy-metrology experiments is therefore not an artifact of probe choice, estimator construction, or measurement strategy. Rather, it is the manifestation of a channel-limited optimization: increasing coherent depth beyond a channel-determined optimum inevitably converts coherent amplification into decoherence-induced information loss, independent of architecture.

By isolating noise into a single retention function, this work provides a clear and platform-independent design principle: *precision is limited not by how much coherence one attempts to build, but by how rapidly the channel destroys it*. Architectural strategies for redistributing resources under fixed budgets, and for exploiting this channel-limited structure in practical sensor design, are addressed in companion papers.

References

[1] Vittorio Giovannetti, Seth Lloyd, and Lorenzo Maccone. Quantum metrology. *Physical Review Letters*, 96(1):010401, 2006.

[2] Barbara M. Escher, Roberto L. de Matos Filho, and Luiz Davidovich. General framework for estimating the ultimate precision limit in noisy quantum-enhanced metrology. *Nature Physics*, 7(5):406–411, 2011.

[3] Rafał Demkowicz-Dobrzański, Jan Kołodyński, and Madalin Guta. The elusive Heisenberg limit in quantum-enhanced metrology. *Nature Communications*, 3:1063, 2012.

[4] Rafał Demkowicz-Dobrzański, Marcin Jarzyna, and Jan Kołodyński. Quantum limits in optical interferometry. In Emil Wolf, editor, *Progress in Optics*, volume 60, pages 345–435. Elsevier, 2015.

[5] Deepinder Sidhu. Heisenberg-saturation across quantum sensing platforms: A universal law of noise-limited precision, 2025. Condensed conference manuscript.