Distributed Structured Quantum Environments in Photonic Quantum Networks:

Extending Structured Quantum Noise from Links to Quantum Network Elements

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July 19, 2026

Abstract

Structured Quantum Noise (SQN) models environmental interactions as structured quantum dynamics in an enlarged Hilbert space rather than as scalar attenuation alone. Prior work extended SQN to quantum-switched optical networks by assigning structured noise maps to communication links and Bell-state-measurement maps to switching nodes. This paper extends that formulation by treating practical photonic network elements themselves as structured quantum environments. In integrated photonic quantum networks, switches, repeaters, Bell-state-measurement modules, detectors, memories, couplers, interferometers, and control interfaces contain internal optical, material, electromagnetic, and measurement degrees of freedom. These internal degrees of freedom may participate in the network's quantum dynamics and should not always be collapsed into ideal switching maps or scalar loss parameters. We introduce a distributed structured-environment model in which both links and nodes are represented as open quantum subsystems with internal Hilbert spaces, local structured maps, and possible cross-element correlations. The resulting formulation yields a generalized ordered-composition model and a global distributed-environment model. The framework predicts that end-to-end quantum-network behavior may depend on link structure, node structure, memory retention, device topology, and the ordering of link–node interactions, not only on hop count or scalar attenuation. The paper provides the mathematical foundation for studying structured environmental effects in photonic quantum networks and motivates collaborative investigation with quantum networking, integrated photonics, quantum computing, and measurement-science communities.

Keywords: structured quantum noise; quantum networks; integrated quantum photonics; quantum switches; photonic interconnects; quantum repeaters; distributed environments; entanglement swapping; quantum memories; quantum internet

1 Introduction

Quantum-network models often separate propagation effects from switching operations. Communication links are assigned loss, dephasing, or memoryless quantum channels, while switching nodes are represented by idealized Bell-state measurements, feed-forward, and Pauli corrections. This separation is analytically useful and is appropriate when the internal device dynamics of network elements are negligible or can be absorbed into effective channel parameters.

Practical photonic quantum networks, however, are not built from ideal abstract maps. They are built from optical fibers, waveguides, couplers, interferometers, beam splitters, phase shifters, detectors, quantum memories, packaging interfaces, integrated photonic circuits, and classical control electronics. These components contain internal physical degrees of freedom that may interact coherently or incoherently with quantum states propagating through the network.

This paper asks the following question:

> *Should the physical implementation of a quantum switching node be treated as part of the structured quantum environment experienced by networked quantum information?*

The answer developed here is conservative. A switch is not merely "the environment." Rather, the physical implementation of a photonic switching node contains internal degrees of freedom that can contribute to the structured environment. Thus, the environment in a practical quantum network may be distributed across links, switches, repeaters, memories, and measurement devices.

The objective of this paper is to extend the link-level SQN network formulation to a distributed network-element formulation.

2 From Structured Links to Distributed Network Environments

The link-level SQN model represents communication link i by a structured environmental Hilbert space $\mathcal{H}_{E_i}$, an initial environmental state σ_{E_i}, and a joint unitary interaction U_i^{link}. If ρ_i is the transmitted quantum state entering the link, the link output is

$$\mathcal{L}_i(\rho_i) = \mathrm{Tr}_{E_i}\left[U_i^{\mathrm{link}}(\rho_i \otimes \sigma_{E_i})(U_i^{\mathrm{link}})^\dagger\right]. \tag{1}$$

In the previous network formulation, the switch was represented by a Bell-state-measurement-based map $\mathcal{S}_i$, giving an ordered path composition of the form

$$\rho_f^{(N)} = (\mathcal{L}_{N+1} \circ \mathcal{S}_N \circ \mathcal{L}_N \circ \cdots \circ \mathcal{S}_1 \circ \mathcal{L}_1)(\rho_0). \tag{2}$$

Equation (2) treats the links as structured but treats switches as idealized operations. The present paper generalizes this by replacing each ideal switching map $\mathcal{S}_i$ with a physical node map $\mathcal{D}_i$ that includes the internal structured dynamics of the photonic network element:

$$\rho_f^{(N)} = (\mathcal{L}_{N+1} \circ \mathcal{D}_N \circ \mathcal{L}_N \circ \cdots \circ \mathcal{D}_1 \circ \mathcal{L}_1)(\rho_0). \tag{3}$$

Here $\mathcal{D}_i$ includes the ideal switching operation as a limiting case but may also include internal device dynamics, imperfect measurement, memory coupling, and structured environmental interactions.

3 Hilbert-Space Model of a Photonic Switching Node

A physical photonic switching node contains internal subsystems. We write its internal Hilbert space schematically as

$$\mathcal{H}_{D_i} = \mathcal{H}_{\mathrm{wg}} \otimes \mathcal{H}_{\mathrm{bs}} \otimes \mathcal{H}_{\phi} \otimes \mathcal{H}_{\mathrm{det}} \otimes \mathcal{H}_{\mathrm{mem}} \otimes \mathcal{H}_{\mathrm{ctrl}} \otimes \mathcal{H}_{\mathrm{mat}} \otimes \cdots. \tag{4}$$

The factors may represent waveguide modes, beam splitters, phase shifters, detector degrees of freedom, memories, control-induced fields, material modes, thermal modes, packaging-induced modes, or other implementation-specific degrees of freedom.

Let ρ_{in} denote the quantum state entering the node and let σ_{D_i} denote the internal device state. A general device-level interaction is

$$\Omega_i^{\text{in}} = \rho_{\text{in}} \otimes \sigma_{D_i}, \tag{5}$$

$$\Omega_i^{\text{out}} = V_i \Omega_i^{\text{in}} V_i^\dagger, \tag{6}$$

where V_i is the joint node interaction. If the device degrees of freedom are not observed, the node induces the reduced map

$$\mathcal{D}_i(\rho_{\text{in}}) = \text{Tr}_{D_i}\left[V_i(\rho_{\text{in}} \otimes \sigma_{D_i})V_i^\dagger\right]. \tag{7}$$

Equation (7) is the node analogue of the structured link map in Eq. (1). Thus links and switches are treated symmetrically: each is a physical quantum subsystem whose unobserved degrees of freedom may influence the reduced network state.

4 Bell-State Measurement as a Structured Device Instrument

A Bell-state measurement is not only a mathematical projection. In a photonic network it is implemented by physical optical and detection hardware. Let $S_L S_R$ denote the two qubits measured at a switching node. The ideal Bell projectors are

$$\Pi_m = |\Phi_m\rangle\langle\Phi_m|, \qquad m = 1, \ldots, 4. \tag{8}$$

The ideal measurement map for outcome m is

$$\tilde{\rho}_{AB}^{(m)} = \text{Tr}_{S_L S_R}\left[(I_A \otimes \Pi_m^{S_L S_R} \otimes I_B)\rho_{A S_L S_R B}(I_A \otimes \Pi_m^{S_L S_R} \otimes I_B)\right]. \tag{9}$$

To include physical device structure, replace the ideal projection by a device-assisted measurement instrument. Let $M_{m,\ell}^{(i)}$ be the effective measurement Kraus operators for node i, where m labels the logical Bell outcome and ℓ labels unresolved internal device degrees of freedom. The node output becomes

$$\tilde{\rho}_{AB}^{(m)} = \sum_\ell \text{Tr}_{S_L S_R D_i}\left[M_{m,\ell}^{(i)}\left(\rho_{A S_L S_R B} \otimes \sigma_{D_i}\right) M_{m,\ell}^{(i)\dagger}\right]. \tag{10}$$

The probability of outcome m is

$$p_m = \text{Tr}_{AB}\left[\tilde{\rho}_{AB}^{(m)}\right], \tag{11}$$

and the corrected node output is

$$\mathcal{D}_i(\rho) = \sum_m \mathcal{C}_m\left(\tilde{\rho}_{AB}^{(m)}\right), \tag{12}$$

where $\mathcal{C}_m$ denotes the classical-feed-forward Pauli correction associated with outcome m.

The ideal Bell-switching map is recovered when the device Hilbert space is trivial and $M_{m,\ell}^{(i)}$ reduces to the ideal Bell projector.

5 Independent Versus Distributed Structured Environments

There are two qualitatively different models.

5.1 Local Independent Structured Elements

In the independent-element model, each link and each node has its own environment:

$$\mathcal{H}_E^{\mathrm{local}} = \bigotimes_{k=1}^{N+1} \mathcal{H}_{E_k}^{\mathrm{link}} \otimes \bigotimes_{j=1}^{N} \mathcal{H}_{D_j}^{\mathrm{node}}. \tag{13}$$

The initial environmental state factorizes:

$$\sigma_E^{\mathrm{local}} = \left(\bigotimes_{k=1}^{N+1} \sigma_{E_k} \right) \otimes \left(\bigotimes_{j=1}^{N} \sigma_{D_j} \right). \tag{14}$$

The network evolution is then described by the ordered CPTP composition in Eq. (3).

5.2 Correlated Distributed Structured Environment

In a distributed structured environment, the internal degrees of freedom of different network elements may not be independent. For example, clocking, thermal fluctuations, packaging, common optical references, memory synchronization, or shared photonic control fields may introduce correlations. The environment state need not factorize:

$$\sigma_E^{\mathrm{dist}} \neq \left(\bigotimes_{k=1}^{N+1} \sigma_{E_k} \right) \otimes \left(\bigotimes_{j=1}^{N} \sigma_{D_j} \right). \tag{15}$$

The total environmental Hilbert space is still

$$\mathcal{H}_E^{\mathrm{dist}} = \left(\bigotimes_{k=1}^{N+1} \mathcal{H}_{E_k}^{\mathrm{link}} \right) \otimes \left(\bigotimes_{j=1}^{N} \mathcal{H}_{D_j}^{\mathrm{node}} \right), \tag{16}$$

but the state and evolution may contain cross-element correlations.

Let the full ordered unitary for an N-switch path be

$$\mathbb{U}_N = U_{N+1}^{\mathrm{link}} V_N^{\mathrm{node}} U_N^{\mathrm{link}} \cdots V_1^{\mathrm{node}} U_1^{\mathrm{link}}, \tag{17}$$

where U_k^{link} acts on the transmitted state and link environment E_k, while V_j^{node} acts on the relevant network state and node environment D_j.

The global distributed-environment output is

$$\rho_f^{(N)} = \mathrm{Tr}_{E^{\mathrm{dist}}} \left[\mathbb{U}_N \left(\rho_0 \otimes \sigma_E^{\mathrm{dist}} \right) \mathbb{U}_N^{\dagger} \right]. \tag{18}$$

Equation (18) is more general than Eq. (3). If the environment factorizes and no cross-element correlations are retained, the global expression reduces to an ordered composition of local link and node maps. If correlations are retained, the evolution cannot in general be decomposed into independent local CPTP maps.

6 Distributed Composition Theorem

Theorem 1 (Distributed Structured-Environment Composition)**.** *Let a quantum network path contain $N+1$ physical links and N physical switching nodes. If each link and node has an independent internal environment, then the end-to-end reduced state is given by the ordered local composition*

$$\rho_f^{(N)} = (\mathcal{L}_{N+1} \circ \mathcal{D}_N \circ \mathcal{L}_N \circ \cdots \circ \mathcal{D}_1 \circ \mathcal{L}_1)(\rho_0). \tag{19}$$

If the network environments are correlated or retained across multiple elements, then the end-to-end state is instead given by the global distributed expression

$$\rho_f^{(N)} = \mathrm{Tr}_{E^{\mathrm{dist}}}\left[\mathbb{U}_N(\rho_0 \otimes \sigma_E^{\mathrm{dist}})\mathbb{U}_N^\dagger\right], \tag{20}$$

and cannot in general be factorized into the local ordered composition in Eq. (19)*.*

Proof. For independent link and node environments, each physical element induces a completely positive trace-preserving reduced map by tracing over its local internal degrees of freedom. Since the output of one network element is the input to the next, the path output is obtained by ordinary composition of the local maps in their physical order. This yields Eq. (19).

For correlated or retained environments, the environmental state does not factorize as a tensor product of independent local states, or the same environmental degrees of freedom participate in more than one network element. In that case, tracing out an environmental subsystem after each local operation changes the state available to later operations and destroys cross-element correlations. Therefore the local reduced maps are not sufficient to reproduce the full network evolution. The correct evolution is obtained by applying the full ordered unitary to the joint network–environment state and tracing over the distributed environment only after the complete path evolution, giving Eq. (20). □

7 Node–Link Ordering Sensitivity

The extension from ideal switches to structured nodes introduces additional noncommutativity. In the link-only model, switching-order sensitivity is governed by the possible noncommutation of link maps and switching maps:

$$\mathcal{L} \circ \mathcal{S} \circ \mathcal{L} \neq \mathcal{S} \circ \mathcal{L}^2. \tag{21}$$

In the distributed model, the relevant comparison becomes

$$\mathcal{L} \circ \mathcal{D} \circ \mathcal{L} \neq \mathcal{D} \circ \mathcal{L}^2, \tag{22}$$

where $\mathcal{D}$ is a physical device map, not an ideal switch. The corresponding commutator action is

$$[\mathcal{L}, \mathcal{D}](\rho) = \mathcal{L}(\mathcal{D}(\rho)) - \mathcal{D}(\mathcal{L}(\rho)). \tag{23}$$

If there exists a reachable state ρ such that

$$[\mathcal{L}, \mathcal{D}](\rho) \neq 0, \tag{24}$$

then node structure influences the end-to-end network state beyond scalar attenuation.

8 End-to-End Metrics

Given $\rho_f^{(N)}$, performance metrics are computed as in the SQN network formulation.

The fidelity relative to a target state ρ_{tar} is

$$F_N = \left(\mathrm{Tr}\sqrt{\sqrt{\rho_{\mathrm{tar}}}\rho_f^{(N)}\sqrt{\rho_{\mathrm{tar}}}} \right)^2 . \tag{25}$$

For a two-qubit endpoint state, concurrence is

$$C_N = \max\left\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\right\}, \tag{26}$$

where the λ_j are the square roots of the eigenvalues of

$$\rho_f^{(N)}(\sigma_y \otimes \sigma_y)(\rho_f^{(N)})^*(\sigma_y \otimes \sigma_y)$$

in nonincreasing order.

Negativity is

$$\mathcal{E}_N = \frac{\left\| (\rho_f^{(N)})^{T_B} \right\|_1 - 1}{2}. \tag{27}$$

If a parameter ϕ is encoded before network propagation, then the output family is

$$\rho_f^{(N)}(\phi) = \mathrm{Tr}_{E^{\mathrm{dist}}}\left[\mathbb{U}_N(\rho_0(\phi) \otimes \sigma_E^{\mathrm{dist}})\mathbb{U}_N^\dagger \right]. \tag{28}$$

The Quantum Fisher Information is

$$F_Q^{(N)} = 2\sum_{a,b} \frac{\left| \langle a_N | \partial_\phi \rho_f^{(N)}(\phi) | b_N \rangle \right|^2}{p_a^{(N)} + p_b^{(N)}}, \tag{29}$$

where $\rho_f^{(N)} = \sum_a p_a^{(N)} |a_N\rangle\langle a_N|$, and terms with zero denominator are omitted.

9 Predictions

The distributed structured-environment model leads to several testable predictions.

1. **Node-induced structured noise.** Even when communication links are well characterized, internal photonic switching dynamics may produce additional structured environmental effects.

2. **Distributed memory effects.** If environmental correlations persist across multiple network elements, end-to-end behavior cannot be reproduced by independent local channel composition.

3. **Node–link ordering sensitivity.** The network state may depend on the physical order of structured link maps and structured node maps.

4. **Metric-dependent degradation.** Fidelity, concurrence, negativity, and QFI may respond differently to structured node and link environments.

5. **Integrated-photonic topology dependence.** Programmable interferometer meshes, detector layouts, phase-shifter networks, and memory interfaces may influence end-to-end quantum information evolution through their internal degrees of freedom.

6. **Timing and recurrence windows.** If internal node or link environments retain coherence, recurrence windows may influence optimal entanglement swapping and repeater scheduling.

10 Relevance to NSF-Style Research Program

This formulation supports a broader research program on the mathematical foundations of structured quantum dynamics. The central NSF-level research questions are:

1. What is the mathematical structure of distributed quantum environments in photonic quantum networks?
2. When can a distributed quantum environment be reduced to independent local quantum channels?
3. What spectral, symmetry, or invariant structures determine recurrence and robustness?
4. How do internal photonic-network degrees of freedom influence fidelity, entanglement, and QFI?
5. Can distributed structured-environment signatures be experimentally detected or benchmarked?

The work is naturally interdisciplinary, combining quantum information theory, open quantum systems, integrated quantum photonics, quantum networking, and measurement science.

11 Conclusion

This paper extends the SQN quantum-network framework from structured communication links to distributed structured quantum environments spanning both links and network nodes. The central conclusion is that practical photonic quantum networks should not always be modeled as ideal switches connected by noisy links. The physical switching elements themselves contain internal quantum and photonic degrees of freedom that may participate in the structured environment experienced by networked quantum information.

The resulting framework replaces the link-only ordered composition

$$\mathcal{L} \circ \mathcal{S} \circ \mathcal{L}$$

with a distributed physical composition

$$\mathcal{L} \circ \mathcal{D} \circ \mathcal{L},$$

and, in the presence of cross-element correlations, with a global distributed-environment evolution

$$\rho_f^{(N)} = \mathrm{Tr}_{E^{\mathrm{dist}}}[\mathbb{U}_N(\rho_0 \otimes \sigma_E^{\mathrm{dist}})\mathbb{U}_N^\dagger].$$

This formulation provides a mathematical foundation for studying structured environmental effects in integrated photonic quantum networks, quantum repeaters, and quantum-switched optical systems. It also provides a natural bridge from the SQN network paper to a broader NSF-style research agenda on distributed structured quantum dynamics.

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